

THEORETICAL MODELING OF OPERATION OF A GAS-OIL WELL UNDER COMPLICATED CONDITIONS

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Two types of solid deposits are usually observed during operation of oil and gas-oil wells. The first type is gas hydrates. At present the concept of the formation of hydrate deposits on well walls has been established [1]. According to this concept, the hydrate-formation intensity is mainly limited by the heat-balance conditions between the hydrate layer, the flow of carbon-hydrogen mixtures in the well, and the surrounding rocks. It is assumed that water and light carbon-hydrogen gases which are necessary for the formation of gas hydrates are always present in excess.

The second type of deposits is high-molecular-weight carbon-hydrogen systems, which hereafter are called paraffin deposits. Separation of paraffin hydrocarbon from oil is caused by the decrease in temperature as the oil rises, gas-phase separation from oil, reduced solubility of the oil, etc. [2]. Under conditions of Western Siberia where rocks at great depths are frozen water-saturated media, the formation of paraffin is much more intense owing to a considerable cooling of the gas-liquid flow moving in a borehole.

Therefore, in analyzing the processes that occur in oil and gas-oil wells during their exploitation, one should take into account three interrelated factors: hydro- and thermal dynamics of liquid-gas flow, solid-phase (paraffin) deposition on the inner walls of the well, and heat exchange between the well and the surrounding rock.

1. Motion of a Liquid-Gas Mixture in a Vertical Channel. To describe the motion of a liquid-gas flow in a well, we make the following basic assumptions: the temperatures in all cross sections of the well are equal in both (gas and liquid) phases, the flow is quasi-steady, and phase transitions occur in equilibrium. To take into account the interphase mass-exchange processes and the associated thermophysical phenomena completely, we also assume that a two-phase mixture consists of three components: a heavy component, a volatile component, and an intermediate component that is present in both the liquid and gas (as vapors) phases. The heavy component, which is a direct precursor of paraffin hydrocarbon deposition on the well walls, can be in the suspended solid phase (paraffin "frog" or "flakes") and in a dissolved state. In addition, the gas is assumed to be calorifically perfect and the fluid is assumed to be incompressible:

$$p = \rho_i^0 R_g T, \quad \rho_i^0 = \text{const.}$$

Here ρ_i^0 ($i = l, g$) is the true phase density; p and T are the pressure and temperature of the two-phase flow; R_g is a gas constant, and the subscripts l and g are referred to the liquid and gas phases, respectively.

The z axis is directed vertically upward, and its origin coincides with the bottom of a well.

Allowing for the above assumptions and ignoring the change in the flow mass which is due to solid-phase deposition, we write an equation of conservation of mass in the form [3]

$$m_l + m_g = m = \text{const}; \tag{1.1}$$

$$k_{l(g)} m_l + k_{g(g)} m_g = m_{(g)}, \tag{1.2}$$

where m_i ($i = l, g$) is the mass rate of the flow in the i th phase through the borehole cross section with the z coordinate and $k_{l(g)}$ and $k_{g(g)}$ are the mass concentrations of the volatile component in the liquid and gas

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phases, respectively. Note that Eqs. (1.1) and (1.2) assume that the flow rates of the entire mixture and of the volatile component, respectively, remain unchanged.

In a steady-flow approximation, the dynamics equation for the entire mixture can be written as

$$m_l \frac{d(v_l^u)}{dz} + m_g \frac{d(v_g^u)}{dz} = -S \frac{dp}{dz} + J^{lg}(v_l^u - v_g^u) - f_w - J^{ls}(v_{ls} - v_l^u) - (\rho_l^0(1 - \alpha) + \rho_g^0\alpha)Sg. \quad (1.3)$$

Here v_i^u ($i = l, g$) is the average-momentum velocity of the i th phase, S is the cross-sectional area of the borehole, f_w is the friction between the flow and the channel wall; J^{lg} and J^{ls} are the gas-emission and solid-deposition intensities per unit length of the channel; v_{ls} is the velocity of the heavy component relative to the liquid phase; α is the volume fraction of the gas, and g is the acceleration of gravity.

During the exploitation of oil wells, the velocity of a liquid-gas mixture is, as a rule, much lower in comparison with the sound velocity in the corresponding mixture. This allows us to ignore terms on the left-hand side of Eq. (1.3) which are attributed to inertial effects. In addition, it is possible to ignore the "reactive" forces $J^{ls}(v_{ls} - v_l^u)$ and $J^{lg}(v_l^u - v_g^u)$ which are related to phase transitions. Equation (1.3) then takes the form

$$\frac{dp}{dz} = -F_w - (\rho_l^0(1 - \alpha) + \rho_g^0\alpha)g \quad (F_w = f_w/S), \quad (1.4)$$

$$F_w = (\lambda_w/4R)(\rho_l^0(1 - \varphi)^2/(1 - \alpha)^2 + \rho_g^0\varphi^2/\alpha^2)W^2 \quad (R = R_0 - \delta_s),$$

where φ is the volume flow-rate concentration of the gas, λ_w is the friction coefficient between the flow and the channel walls; W is the mean discharge velocity; R_0 is the radius of a lifting column, and δ_s is the paraffin-deposit thickness.

For the slug and circular flow regimes, to find the friction coefficient, we can employ the relation [4]

$$\lambda_w = 0.067(158/\text{Re} + \varepsilon/R)^{0.2} \quad [\text{Re} = (2\rho_l^0(1 - \alpha)WR)/\mu_l],$$

where μ_l is the dynamic liquid viscosity and ε is the tube roughness.

For the volume concentration of the gas, we use the relation proposed by A. A. Armand [5]:

$$\alpha = \begin{cases} 0.833\varphi, & \varphi \leq 0.9, \\ [0.833 + 0.167(1 + \rho_l^0(1 - \varphi)/(\rho_g^0\varphi))^{-1}]\varphi, & \varphi > 0.9 \end{cases} \quad \left(\varphi = \left[1 + \frac{\rho_g^0 m_l}{\rho_l^0 m_g}\right]^{-1}\right).$$

The equation for heat influx has the form

$$mc \frac{dT}{dz} = \frac{m_g}{\rho_g^0} \frac{dp}{dz} + m_l(l_{(g)}^{lg} - l_{(l)}^{lg}) \frac{dk_{l(g)}}{dz} + [l_{(g)}^{lg}k_{g(g)} + l_{(l)}^{lg}(1 - k_{l(g)})] \frac{dm_l}{dz} - Q_w, \quad (1.5)$$

where $mc = m_l c_l + m_g c_g$; $Q_w = 2\pi R q_w$; c_i ($i = l, g$) is the specific heat of the i th phase; $l_{(l)}^{lg}$ and $l_{(g)}^{lg}$ are, respectively, the specific heat of liquid vaporization and solvent-gas separation; and Q_w is the heat-outflux intensity per unit length of the well.

From (1.1) and (1.2), it follows that

$$\frac{dm_l}{dz} = \left(m_l \frac{dk_{l(g)}}{dz} + m_g \frac{dk_{g(g)}}{dz}\right) / (k_{g(g)} - k_{l(g)}). \quad (1.6)$$

Substituting (1.6) into (1.5), we have

$$mc \frac{dT}{dz} = \frac{m_g}{\rho_g^0} \frac{dp}{dz} + A_l \frac{dk_{l(g)}}{dz} + A_g \frac{dk_{g(g)}}{dz} - Q_w, \quad A_l = m_l(l_{(g)}^{lg} - l_{(l)}^{lg}) + \frac{l_{(g)}^{lg} m_l}{k_{g(g)} - k_{l(g)}}, \quad (1.7)$$

$$A_g = l_{(g)}^{lg} m_l / (k_{g(g)} - k_{l(g)}), \quad l_{(g)}^{lg} = l_{(g)}^{lg} k_{g(g)} + l_{(l)}^{lg} (1 - k_{l(g)}).$$

We also assume that the dependence of the partial pressure of the volatile component on the mass concentration of this component in the liquid phase obeys Henry's law:

$$p_{(g)} = G_{(g)} k_{l(g)}, \quad (1.8)$$

where $G_{(g)}$ is Henry's constant.

Let us set the partial pressure of the liquid vapors (intermediate component) equal to the saturation pressure at a given temperature. The pressure dependence on temperature is usually written as

$$p_{(l)} = p_{(l)}(T) = p_{(l)*} \exp(-T_*/T), \quad (1.9)$$

where $p_{(l)*}$ and T_* are empirical approximating parameters. In addition, the gas-phase pressure is assumed to satisfy Dalton's law:

$$\begin{aligned} p &= p_{(l)} + p_{(g)}, \quad p_{(l)} = \rho_{g(l)}^0 R_{(l)} T, \quad p_{(g)} = \rho_{g(g)}^0 R_{(g)} T, \\ \rho_{g(l)}^0 + \rho_{g(g)}^0 &= \rho_g^0, \quad R_{(l)}(1 - k_{g(g)}) + R_{(g)} k_{g(g)} = R_g, \end{aligned} \quad (1.10)$$

where $\rho_{g(l)}^0$ and $\rho_{g(g)}^0$ and $R_{(l)}$ and $R_{(g)}$ are the real densities and the gas constants of the intermediate and volatile components, respectively, in the gas phase.

Using relations (1.8)–(1.10) and excluding $p_{(l)}$ and $p_{(g)}$, we obtain

$$p = p_{(l)}(T) + G_{(g)} k_{l(g)}; \quad (1.11)$$

$$\frac{p_{(l)}(T)}{p} = \frac{R_{(l)}(1 - k_{g(g)})}{R_{(l)}(1 - k_{g(g)}) + R_{(g)} k_{g(g)}} = B. \quad (1.12)$$

Differentiating (1.11) and (1.12) with respect to the z coordinate, we have

$$\begin{aligned} \frac{dp}{dz} &= \frac{dp_{(l)}(T)}{dz} + G_{(g)} \frac{dk_{l(g)}}{dz}, \quad \frac{dp_{(l)}(T)}{dz} = p'_{(l)}(T) \frac{dT}{dz} = B \frac{dp}{dz} + B' p \frac{dk_{g(g)}}{dz}, \\ p'_{(l)}(T) &= \frac{dp_{(l)}(T)}{dT} = \frac{T_* p_{(l)}(T)}{T^2}, \quad B' = \frac{dB}{dk_{g(g)}} = -\frac{R_{(l)} R_{(g)}}{(R_{(l)}(1 - k_{g(g)}) + R_{(g)} k_{g(g)})^2} \end{aligned}$$

from which

$$\frac{dk_{l(g)}}{dz} = \left(\frac{dp}{dz} - p'_{(l)}(T) \frac{dT}{dz} \right) / G_{(g)}; \quad (1.13)$$

$$\frac{dk_{g(g)}}{dz} = \left(p'_{(l)}(T) \frac{dT}{dz} - B \frac{dp}{dz} \right) / B' p. \quad (1.14)$$

According to the differential equations (1.4), (1.7), (1.13), and (1.14), finding the distribution of the thermodynamic parameters over a well requires specifying the boundary conditions at the well bottom and its head. As one of them, it seems reasonable to require that the pressure at the wellhead in the operation well be constant. In addition, it is assumed that the flow rate of the well is determined by the oil-reservoir and hole-bottom pressures. As the simplest dependence of this kind, we use the expression $m = K(p_b - p_f)$, where p_b and p_f are the oil-reservoir and hole-bottom pressures.

The parameter K , which is a characteristic of the oil reservoir, is assumed to be constant. The value of K can be found, depending on the operational mode of the well (using the known quantities p_b , p_f , and m). We also assume that the hole-bottom T_f and reservoir temperatures are equal. The concentrations of the volatile component in the liquid and gas phases [$k_{l(g)f}$ and $k_{g(g)f}$] in the hole bottom are found from relations (1.11) and (1.12) via the bottom pressure and temperature:

$$k_{l(g)f} = (p_f - p_{(l)}(T_f)) / G_{(g)}; \quad (1.15)$$

$$p_{(l)}(T_f) / p_f = R_{(l)}(1 - k_{g(g)f}) / (R_{(l)}(1 - k_{g(g)f}) + R_{(g)} k_{g(g)f}). \quad (1.16)$$

With the pressure and the temperature at the bottom assigned and with the concentrations $k_{l(g)f}$ and $k_{g(g)f}$ known from (15) and (16), we determine the distribution of the hydrodynamic parameters over the well and at its head by integrating numerically system (1.4), (1.7), (1.13), and (1.14).

We can reach a constant pressure at the wellhead by choosing an appropriate pressure at the bottom of the well. The wellhead pressure has an effect on the temperature situation in the lifting tubing. Figure 1 shows the curve of the mean temperature of the gas-liquid flow versus the wellhead pressure at various

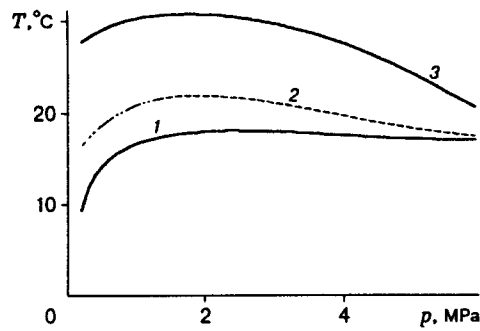


Fig. 1

depths (curves 1-3 refer to $h = 0, 200,$ and 500 m). Here and below, the parameters which were used to calculate the properties of the system considered are as follows: $p_b = 22.7$ MPa, $p_f|_{t=0} = 13.6$ MPa, $T_f = 353$ K, $H = 3000$ m, $R_0 = 0.0315$ m, $R_w = 0.25$ m, $m_l = 0.463$ kg/sec, $m_g = 0.083$ kg/sec, $\rho_l^0 = 850$ kg/m³, $\rho_s^0 = 900$ kg/m³, $\mu_l = 4.1 \cdot 10^{-3}$ kg/(m·sec), $\mu_g = 1.25 \cdot 10^{-5}$ kg/(m·sec), $R_{(g)} = 520$ m²/(sec²·K), $R_{(l)} = 52$ m²/(sec²·K), $\lambda_l = 0.13$ kg·m/(sec³·K), $\lambda_s = 0.27$ kg·m/(sec³·K), $c_l = 2100$ m²/(sec³·K), $c_g = 2500$ m²/(sec³·K), $l_{(l)}^{lg} = 2.3 \cdot 10^5$ m²/sec², $l_{(g)}^{lg} = 10^4$ m²/sec², $l^s = 2 \cdot 10^5$ m²/sec², and $\varepsilon = 1.5 \cdot 10^{-5}$ m. Here H is the depth of the well; ρ_s^0 , λ_s , and l^s are the true density, the specific thermal conductivity, and the crystallization heat of paraffin, respectively, and R_w is the outer radius of the well. We assume that there is a fluid (oil) in the borehole and, for convenience of calculations, the temperature at the external boundary of the well is equal to the geothermal $T_0^{(1)}$.

As seen in Fig. 1, at a definite pressure at the wellhead, the temperatures in the borehole are the highest ones. In our opinion, this can be accounted for as follows.

Analysis of Eq. (1.7) shows that there are three basic factors for the change in the mean temperature of the liquid-gas flow as it rises to the wellhead: adiabatic expansion, phase transitions, and heat outflux to the surrounding rock. Note that the temperature variation that is caused by phase transitions can be ignored. With a decrease in the wellhead pressure, the pressure at the well bottom also reduces, and the flow rate grows. As a consequence, moving more rapidly to the wellhead (the velocities of the mixture components increase), the product of the well releases a smaller portion of heat to the surrounding rock. However, with the convective heat exchange reduced, the losses in temperature by the gas-liquid flow, which are due to adiabatic expansion, grow. Thus, it is the realization of these factors that is responsible for the highest temperatures in the borehole at a certain pressure at the wellhead. A similar nonmonotonic dependence of the temperature on the pressure at the exit from the pipe in the flow of a real and actual gas is noted in [6].

To close Eqs. (1.4), (1.7), (1.13), and (1.14), it is necessary to give the heat outflux intensity Q_w and also the paraffin-deposition intensity on the inner walls of the well.

2. Kinetics of Paraffin Deposition. For the formation of paraffin deposits, it is necessary that the carbon-hydrogen mixture contain heavy (high-molecular-weight) components, and the temperature regime in the well (especially in the vicinity of its walls) admit the existence of the heavy phase.

In the general case, a narrowing of the transverse cross section ("sclerosis") of the borehole, which is caused by paraffin deposits, is accompanied by the formation of solid-phase condensed particles in the liquid layer flowing near the walls. The formation of solid particles occurs owing to overcooling of this layer below the crystallization temperature of the heavy carbon-hydrogen components and further transfer of these particles to the channel wall (for example, under the action of thermocapillary forces). In addition, the diffusive and thermodiffusive transfers of heavy carbon-hydrogen components to the channel wall take part in the formation of the solid phase and its growth.

We assume that the formation of the solid phase (paraffin) is mainly determined by the fact that the fluid loses the ability to flow near the walls because of its freezing with decreasing temperature. This assumption means that the fluid contains in excess heavy components producing the solid phase, and the

solid-phase deposition intensity J^{ls} is found from the condition of the heat balance on the solid-deposit surface:

$$\rho_s^0 l^s \frac{\partial \delta_s}{\partial t} = -q_w - \lambda_s \left(\frac{\partial T_s}{\partial r} \right)_R. \quad (2.1)$$

We also assume that the inner surface of the solid phase is the isothermal surface whose temperature T_σ is equal to the crystallization temperature T_e in a carbon-hydrogen mixture flowing in the well.

The intensity of heat exchange between the solid-phase surface and the flow is assumed to have the form

$$q_w = \beta_w (T - T_\sigma), \quad (2.2)$$

where β_w is the convective heat-transfer coefficient, which depends on the structure of the liquid-gas flow in the well and also on the specific features of the flow in the near-surface layer in the vicinity of the solid-phase or the channel walls.

Since the paraffin layer grows rather slowly on the well wall (i.e., the characteristic time of complete clogging of the well is much greater than that for which the stationary temperature profile inside the solid phase is established: $t_s = \delta_s^2 / \chi_s$, where χ_s is the thermal diffusivity of paraffin), we assume that at each moment, the distribution of the temperature T_s in the solid layer satisfies the equation

$$r^{-1} \frac{\partial}{\partial r} \left(\chi_s r \frac{\partial T_s}{\partial r} \right) = 0 \quad (R < r < R_0).$$

Having found the temperature distribution over the solid layer from the above equation, we substitute both the quantity obtained and (2.2) into Eq. (2.1). As a result, we obtain

$$\rho_s^0 l^s \frac{\partial \delta_s}{\partial t} = -\beta_w (T - T_\sigma) + \lambda_s \frac{T_0 - T_\sigma}{R \ln(R/R_0)}. \quad (2.3)$$

Here T_0 is the temperature of the inner wall of the well. To find it, the problem on heat exchange of a well with the surrounding rock should be considered.

The above pattern of solid-phase deposition assumes that the intensity of this process can be found using the heat-balance condition. In the more general case, the deposition intensity depends on the mass transfer of the heavy component to the solid-phase surface. For the solid-phase formation intensity, we then write the relation

$$j_{w(s)} = \frac{2\rho_l^0 D}{R(1-\alpha)} (k_{l(s)} - k_{l(s)e}), \quad \frac{\partial \delta_s}{\partial t} = \frac{j_{w(s)}}{\rho_s^0},$$

where $j_{w(s)}$ is the intensity of mass transfer of the heavy component per unit area of the solid phase, D is the diffusion coefficient, and $k_{l(s)}$ and $k_{l(s)e}$ are, respectively, the mean-mass concentration of the heavy component in the fluid and the equilibrium concentration of the heavy component at a temperature equal to the temperature of the solid-phase surface. Using the Schroeder equation, for the dependence of $k_{l(s)e}$ on the temperature of the solid-phase surface, we can use the expression $k_{l(s)e} = k_{l(s)e}^* \exp(-T_s^*/T_\sigma)$, where $k_{l(s)e}^*$ and T_s^* are empirical approximating parameters.

If one assumes that no depletion of the heavy component [$k_{l(s)} = \text{const}$] occurs and the temperature of the solid-phase surface in the separation zone changes slightly, then the dependence of $k_{l(s)e}$ on T_σ can be regarded as a linear dependence. The relation for the solid-phase formation intensity then takes the form

$$j_{w(s)} = K_m (T_e - T_\sigma), \quad K_m = \frac{2\rho_l^0 D}{R(1-\alpha)} \left(\frac{\partial k_{l(s)e}}{\partial T_\sigma} \right)_{T_e}. \quad (2.4)$$

Excluding the value of the temperature T_σ on the solid-phase surface from relations (2.3) and (2.4), we obtain

$$\frac{\partial \delta_s}{\partial t} = \frac{\beta_w (T_e - T) + \lambda_s (T_0 - T_e) / (R \ln(R/R_0))}{\rho_s^0 l^s + (\beta_w - \lambda_s / (R \ln(R/R_0))) / K_m}.$$

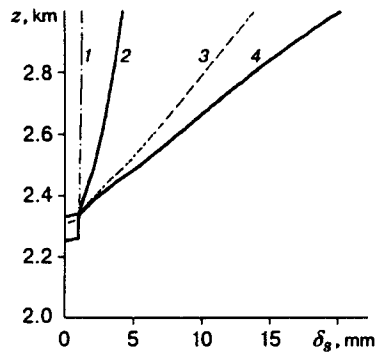


Fig. 2

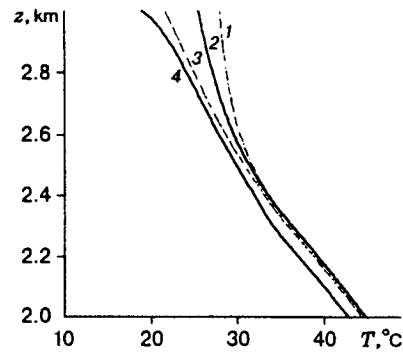


Fig. 3

The value of the temperature on the solid-phase surface is found from the expression

$$T_{\sigma} = \frac{\beta_w T + K_m l^s T_e - \lambda_s T_0 / (R \ln(R/R_0))}{\beta_w + K_m l^s - \lambda_s / (R \ln(R/R_0))}.$$

On the basis of the proposed pattern, we constructed the profiles of paraffin deposition on the inner wall of the lifting tubing at various moments after the beginning of the operation of the well (curves 1–4 in Fig. 2 correspond to $t = 1, 4, 7,$ and 8 days). Figure 3 shows the mean-temperature distribution of liquid–gas flow over the upper section of the well with variation in the thickness of paraffin deposits, whose profiles are depicted in Fig. 2. In calculations, the temperature at the beginning of paraffin crystallization is assumed to equal 30°C .

As seen in Figs. 2 and 3, paraffin deposits not only have no heat-insulating action but, on the contrary, even intensify the process of paraffin formation. In our opinion, this occurs for the following reason. As the deposit thickness increases, the temperature in the borehole drops owing to an increase in well pressure and a decrease in flow rate (and the related change in phase velocities). In turn, the temperature drop contributes to a more rapid growth of paraffin deposition.

3. Heat Exchange of a Well with the Surrounding Rock. To specify the heat-exchange intensity between a well and the surrounding rock, we shall consider, in sequence, the heat-exchange intensity between the flow and the well wall (or the solid phase on the wall), the heat flux through the walls, and the heat transfer to the surrounding rock.

Let us divide the well into two sections along its length. In the first section which lies between the hole bottom and the cross section of the well where the temperature of the inner wall reaches the crystallization temperature for the heavy component, no solid phase is present. In this case, for the heat-exchange intensity we write $q_w = \beta_w(T - T_0)$.

To find the inner-wall temperature T_0 , it is necessary to consider the problem of external heat exchange. Note that if the cross section where T_0 reaches the crystallization temperature for the heavy component is above the floor of the frozen rocks, in defining the temperature of the inner wall of the well the melting region which forms around the well should be taken into account.

The second section is between the cross section where the solid phase starts forming and the wellhead. In this paraffin-deposition area of the well, the heat-exchange intensity has the following form: $q_w = \beta_w(T - T_{\sigma})$.

In analysis of the unsteady heat exchange from the inner wall of the borehole to the surrounding rocks [7] through which the well was drilled, it is usually assumed that heat exchange is described using the coefficient

$$\beta = \frac{1}{R_N \sum_{i=1}^N \lambda_i^{-1} \ln(R_i/R_{i-1})}, \quad R_N = R_w. \quad (3.1)$$

In the above expression, R_i is the outer radius of the i th layer and λ_i is the thermal conductivity in this layer;

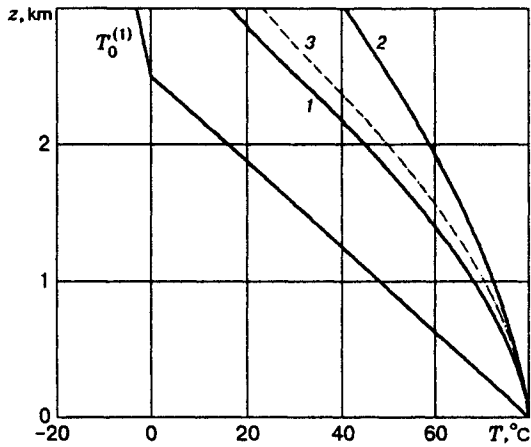


Fig. 4

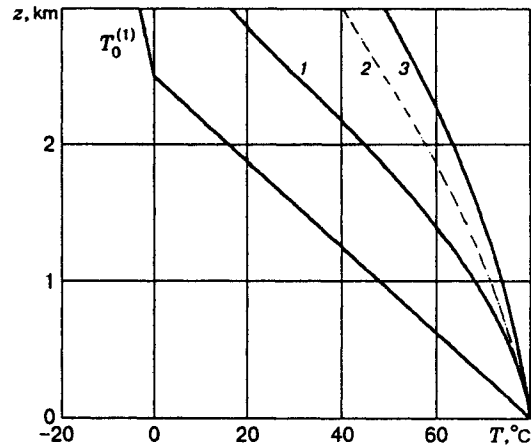


Fig. 5

note that $i = 1$ corresponds to the tubing, and $i = N$ corresponds to the layer adjacent to the surrounding rock.

If the medium in the interpipe space is in a thermogravitational-convection state, the thermal conductivity of the medium should be multiplied by the correction [7]

$$\alpha = 0.049 (\text{Gr Pr})^{1/3} \text{Pr}^{0.074},$$

where Gr and Pr are the Grashof and Prandtl numbers, respectively.

Figure 4 shows the distribution of the mean temperature of the flow over the well, depending on the kind and state of a substance in the interpipe space. It is seen that if this space is filled with a substance of lower thermal conductivity, the temperature regime in the borehole substantially improves (curves 1 and 2 correspond to oil and gas, respectively). On the other hand, if the gas is in a thermogravitational-convection state (curve 3), this leads to increasing heat losses of the well. The use of thermally insulated pipes makes it possible to improve the temperature situation in the borehole (curves 1–3 in Fig. 5) correspond to insulating-material thicknesses of 0, 5, and 10 mm (the remaining volume of the interpipe space is filled with oil). However, the fabrication of such equipment is rather complicated and, consequently, pipes of this kind are very expensive. We performed calculations for polyurethane [coefficient of thermal conductivity, $\lambda = 0.0067 \text{ W}/(\text{m} \cdot \text{K})$] as an insulating material.

As previously mentioned, to find the temperature T_0 of the inner wall, the interaction between the well and the surrounding rock should be analyzed. If oil and gas–oil wells operate under conditions of permafrost congealment, at definite depths this interaction is accompanied by melting the ground in the vicinity of the wells. Therefore, in the general case, to describe the external heat exchange of a well, it is necessary to solve the problem of thermal conductivity with allowance for phase transitions.

Prior to the moment of appearance of melted rock $t^{(1)}$, heat transfer between the well and the surrounding rock in the area which lies below the floor of the frozen rocks or within them obeys the equation of thermal conductivity

$$\frac{\partial T^{(1)}}{\partial t} = \chi^{(1)} r^{-1} \frac{\partial}{\partial r} \left(r \frac{\partial T^{(1)}}{\partial r} \right), \quad 0 < t < t^{(1)}, \quad r > R_w, \quad \chi^{(i)} = \lambda^{(i)} / \rho^{(i)} c^{(i)}. \quad (3.2)$$

Here and below, $\rho^{(i)}$, $c^{(i)}$, $T^{(i)}$, $\lambda^{(i)}$, and $\chi^{(i)}$ are the density, specific heat, temperature, thermal conductivity, and thermal diffusivity, respectively; the superscript in the brackets refers to the parameters of the frozen ($i = 1$) and melted ($i = 2$) rocks.

At the well-rock interface, we assume that

$$-\lambda^{(1)} \frac{\partial T^{(1)}}{\partial r} = \beta(T_0 - T^{(1)}), \quad 0 < t < t^{(1)}, \quad r = R_w. \quad (3.3)$$

At infinity, we require that the limited-temperature condition be satisfied: $\partial T^{(1)}/\partial r = 0$, $0 < t < t^{(1)}$, and $r = \infty$. In addition, we shall write the equality condition for thermal fluxes through the inner and outer walls of the well:

$$\lambda^{(1)} R_w \left(\frac{\partial T^{(1)}}{\partial r} \right)_{R_w} = \lambda_s R_0 \left(\frac{\partial T_s}{\partial r} \right)_{R_0}. \quad (3.4)$$

For the processes considered, the external-heat problem described above is solved rather accurately and effectively by the integral method [8, 9], according to which the temperature distribution around a well has the form

$$T^{(1)} = C_1 \ln(r/R_w) + C_2(r/R_w) + C_3, \quad (3.5)$$

subject to the boundary conditions

$$T^{(1)} = T_0^{(1)}, \quad \partial T^{(1)}/\partial r = 0, \quad r = R_*(t), \quad (3.6)$$

where $R_*(t)$ is the radius of the thermal action of the well, the coefficients C_1 , C_2 , and C_3 are found from the condition under which function (3.5) is subject to the boundary conditions (3.3) and (3.6), and R_* is determined using (3.2).

We can derive from relation (3.4) an expression for the temperature on the inner surface of the wall, depending on both the thickness of solid deposits and the radius of action of the well:

$$T_0 = T_\sigma - \frac{B_1(T_0^{(1)} - T_\sigma) \ln(R/R_0)}{\lambda_s/\lambda^{(1)} - B_1 \ln(R/R_0)}, \quad B_1 = \frac{\beta^{(1)}(\theta_* - 1)}{\theta_* - 1 + \beta^{(1)}(\theta_* \ln(\theta_*) - \theta_* + 1)},$$

$$\beta^{(i)} = \beta R_w / \lambda^{(i)} \quad (i = 1, 2), \quad \theta_* = R_*/R_w.$$

The relations obtained determine the heat distributions around the well until the moment when the rock temperature on the borehole wall reaches the melting point $T^{(1,2)}$ of the frozen rock. To describe the subsequent process of heat exchange between the well and the rock, we should take into account the presence of the melted zone between the well and the frozen-rock surface of radius $R^{(1,2)}$. In accordance with this, we write the external heat problem around the well as follows:

$$\frac{\partial T^{(2)}}{\partial t} = \chi^{(2)} r^{-1} \frac{\partial}{\partial r} \left(r \frac{\partial T^{(2)}}{\partial r} \right), \quad t > t^{(1)}, \quad R_w < r < R^{(1,2)}; \quad (3.7)$$

$$\frac{\partial T^{(1)}}{\partial t} = \chi^{(1)} r^{-1} \frac{\partial}{\partial r} \left(r \frac{\partial T^{(1)}}{\partial r} \right), \quad t > t^{(1)}, \quad R^{(1,2)} < r < \infty; \quad (3.8)$$

$$\lambda^{(2)} \frac{\partial T^{(2)}}{\partial r} = \beta(T^{(2)} - T_0), \quad t > t^{(1)}, \quad r = R_w; \quad (3.9)$$

$$T^{(1)} = T^{(2)} = T^{(1,2)}, \quad r = R^{(1,2)}; \quad (3.10)$$

$$-\lambda^{(2)} \frac{\partial T^{(2)}}{\partial r} + \lambda^{(1)} \frac{\partial T^{(1)}}{\partial r} = \rho^{(1)} l^{(1)} \frac{dR^{(1,2)}}{dt}, \quad t > t^{(1)}, \quad r = R^{(1,2)}, \quad \frac{\partial T^{(1)}}{\partial r} = 0, \quad r = \infty, \quad (3.11)$$

where $l^{(1)}$ is the heat of melting of the frozen rock.

By analogy with the previous case, we use the $T^{(1)}$ distribution in the form of (3.5). The coefficients C_1 , C_2 , and C_3 are found using the boundary conditions (3.6) and (3.10), and $R_*(t)$ is found by Eqs. (3.8) and (3.10).

We determine the temperature profiles in the melted zone by a successive choice of steady states [10.

11] under the condition that the temperature distribution satisfies the equation

$$r^{-1} \frac{\partial}{\partial r} \left(r \frac{\partial T^{(2)}}{\partial r} \right) = 0 \quad (R_w < r < R^{(1,2)}).$$

With allowance for the boundary conditions (3.9) and (3.10), the solution of the above expression has the form

$$T^{(2)} = T^{(1,2)} + \frac{\beta^{(2)}(T_0 - T^{(1,2)}) \ln(\theta^{(1,2)} R_w r^{-1})}{1 + \beta^{(2)} \ln(\theta^{(1,2)})}, \quad \theta^{(1,2)} = R^{(1,2)} / R_w.$$

The radius $R^{(1,2)}$ of the melted zone is found from Stefan's condition (3.11). Taking into account, as in the preceding case, that

$$\lambda^{(2)} R_w \left(\frac{\partial T^{(2)}}{\partial r} \right)_{R_w} = \lambda_s R_0 \left(\frac{\partial T_s}{\partial r} \right)_{R_0},$$

we write the relation

$$T_0 = T_\sigma + \beta^{(2)}(T^{(1,2)} - T_\sigma) \ln(R/R_0) / \{ \beta^{(2)} \ln(R/R_0) - \lambda_s(1 + \beta^{(2)} \ln(\theta^{(1,2)})) / \lambda^{(2)} \}$$

for the temperature at the inner surface of the channel.

Thus, we have developed a theoretical model of operation of a well with allowance for the combined action of processes such as the flow of a gas-oil mixture in the borehole of a well, paraffin deposition on the inner wall of the channel, and heat exchange between the well and the surrounding frozen rock.

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